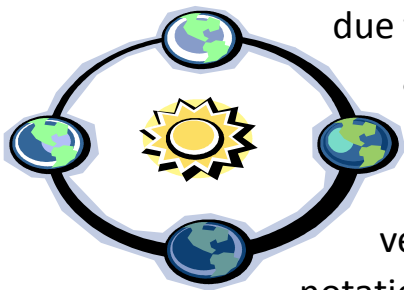

SCIENTIFIC NOTATION

You might read that the distance from the Earth to the Sun is approximately 9.3×10^7 miles.

[The distance varies somewhat throughout the year due to the Earth's elliptical (oval) orbit around the sun.] This way of writing a number is called ***scientific notation***, and is used to write very large and very small numbers in a compact notation that's easier to understand than long strings of digits.



□ THE DISTANCE FROM THE EARTH TO THE SUN

To convert 9.3×10^7 (distance from the Earth to the Sun) into a regular number, we need only our knowledge of exponents:

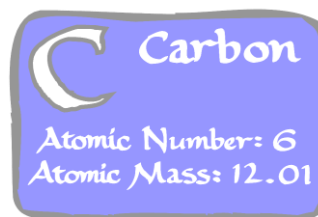
$$\begin{aligned}
 &9.3 \times 10^7 \\
 = &9.3 \times (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \\
 = &9.3 \times 10,000,000 \\
 = &\mathbf{93,000,000}
 \end{aligned}$$

Thus, 9.3×10^7 miles is simply 93 million miles. Why not just write “93 million miles,” or 93,000,000 miles? Because with other numbers it might become quite cumbersome.

If we were to expand, for instance, the number 6.02×10^{23} , we’d get “602” followed by 21 zeros:

$$6.02 \times 10^{23} = \mathbf{602,000,000,000,000,000,000,000}$$

Not only would it be horrendous to write it this way, it would overflow a calculator (or convert it right back to scientific notation!), and most people wouldn't even know the name for a number that big — thus I hope you see the need for scientific notation.



6.02×10^{23} is the number of **atoms** in 12 grams of Carbon-12.

❑ **MULTIPLYING AND DIVIDING BY POWERS OF 10**

Before we begin the actual conversions, let's review how we multiply and divide by powers of 10.

The number 7.06×10^6 **miles** is calculated by moving the decimal point 6 places to the right, resulting in **7,060,000 miles**.

To multiply by a negative power of 10, move the decimal point to the left. For example, 6.09×10^{-5} grams = **0.0000609 grams**.

Can you see how we know whether to move the decimal point to the right or to the left?

❑ **THE DEFINITION OF SCIENTIFIC NOTATION**

A number is in **scientific notation** if it's written in the form

$$M \times 10^C$$

where M is a real number such that $1 \leq M < 10$, and C is an integer (whole numbers and their negatives).

Also notice that the M is being multiplied by the power of **10**. In the Introduction, we stated that the distance from the

$$M \times 10^C$$

M is any number between 1 and 10 (including the 1, but excluding the 10), and C must come from the set of integers:

$\{ \dots -3, -2, -1, 0, 1, 2, 3, \dots \}$

Earth to the Sun was 9.3×10^7 miles, a number written in *scientific notation*.

The following are in scientific notation, and the reason is that in each case:

- 1) the operation is multiplication,
- 2) the front number is between 1 and 10 (possibly 1, but not 10),
- 3) the base is 10, and
- 4) the exponent is an integer.

$$4.56 \times 10^7 \qquad 9.7023 \times 10^{-9} \qquad 8 \times 10^0 \qquad 1 \times 10^{34}$$

The following, however, are not in scientific notation:

23.9×10^{17}	The front number is too big.
0.72×10^{-67}	The front number is too small.
10×10^{44}	The front number is too big.
7.023×2^{34}	The base (the 2) is not 10, as it must be.
$3.4 \times 10^{1/2}$	The exponent is not an integer.
$1.13 + 10^{19}$	It's an addition problem, not multiplication.

Homework

1. Is the given number in scientific notation?

- | | | |
|----------------------------|-----------------------------|----------------------------|
| a. 1.0007×10^{77} | b. 9.999×10^{-5} | c. 13×10^8 |
| d. 0.77×10^{23} | e. $2.34 + 10^7$ | f. $4.5 \times 10^{1.3}$ |
| g. 3.44×2^9 | h. 10.1×10^{-9} | i. 9.99×10^1 |
| j. 2.66×10^0 | k. $4.34^2 \times 10^{-23}$ | l. $2.39 \times 10^{-1/3}$ |

□ CONVERTING WITH SCIENTIFIC NOTATION

EXAMPLE 1: Convert each scientific notation number into a regular number (sometimes called standard notation):

A. 2.73×10^4

Move the decimal point 4 places to the right $\Rightarrow$ **27,300**

B. 8.9×10^1

Move the point 1 place to the right $\Rightarrow$ **89**

C. 1.33×10^0

Move the point 0 places to the right $\Rightarrow$ **1.33**

[Or, since $10^0 = 1$, the answer must be 1.33.]

D. 4.1×10^{-3}

Move the point 3 places to the left $\Rightarrow$ **0.0041**

E. 7.827×10^{-8}

Move the point 8 places to the left $\Rightarrow$ **0.00000007827**

Homework

2. Convert each scientific notation number into a regular number:

- | | | | |
|--------------------------|--------------------------|-------------------------|--------------------------|
| a. 8.73×10^2 | b. 2.09×10^5 | c. 8.0×10^1 | d. 2.36×10^{10} |
| e. 5.99×10^{-2} | f. 6.01×10^{-4} | g. 1.0×10^{-6} | h. 8.35×10^0 |

EXAMPLE 2: Convert each regular number into scientific notation:

A. 540.92

We need the decimal part of the scientific notation number to be between 1 and 10, so we move the decimal point 2 places to the left, giving 5.4092, which is equivalent to dividing by 100. We compensate for this dividing by multiplying the new decimal by 100, or 10^2 . We end up with

$$5.4092 \times 10^2$$

B. 0.000056

To get the decimal in the right spot for scientific notation, we need to move it 5 places to the right, which is essentially multiplying the number by 100,000; this produces 5.6. We balance this maneuver by “dividing” by 100,000, which is equivalent by multiplying by 10^{-5} . Thus, the given decimal is equal to

$$5.6 \times 10^{-5}$$

Homework

3. Convert each regular number into scientific notation:

a. 9,200,000

b. 23,400,000,000

c. 34.56

d. 0.0000123

e. 76,996.7

f. 0.0000000077

EXAMPLE 3: Convert each number into scientific notation:

Note that the following two problems may look vaguely like scientific notation, but they're not.

A. 238.2×10^7

First, be sure you understand why the number is not already in scientific notation. To convert the decimal to a number between 1 and 10 (but not 10), shift the decimal point 2 places to the left (giving 2.382), which is equivalent to dividing by 100. Now balance this by multiplying the power of 10 by 100, which gives $10^7 \times 10^2 = 10^9$. Final result:

$$\mathbf{2.382 \times 10^9}$$

B. 0.0076×10^{-5}

The decimal is not between 1 and 10, so we shift the decimal point 3 places to the right to make it between 1 and 10, so now it's 7.6. That shift is the same as multiplying by 1000, so we compensate by dividing 10^{-5} by 1000, which is accomplished by subtracting 3 from the exponent, giving 10^{-8} . And we're done:

$$\mathbf{7.6 \times 10^{-8}}$$

Homework

4. Convert each number into scientific notation:

a. 23.9×10^5

b. 127.05×10^{14}

c. 0.19×10^{-8}

d. 0.015×10^{-10}

e. 832.44×10^{-7}

f. 0.004×10^{-5}

❑ **MULTIPLYING AND DIVIDING WITH SCIENTIFIC NOTATION**



Important: Notice that in all of the following calculations, you are not allowed to use the EE button on your calculator. Your calculator is to be used only for simple multiplication and division of decimals.

EXAMPLE 4: Calculate the product: $(9.2 \times 10^4)(1.7 \times 10^7)$

Solution: Notice that the problem contains nothing but multiplications; we can therefore rearrange the four factors in any manner we please (associative and commutative properties).

$$\begin{aligned}
 & (9.2 \times 10^4)(1.7 \times 10^7) \\
 = & 9.2 \times 1.7 \times 10^4 \times 10^7 && \text{(rearrangement)} \\
 = & (9.2 \times 1.7) \times (10^4 \times 10^7) && \text{(regroup)} \\
 = & 15.64 \times 10^{11} && \text{(add the exponents)} \\
 = & \mathbf{1.564 \times 10^{12}} && \text{(convert to scientific notation)}
 \end{aligned}$$

EXAMPLE 5: Find the quotient: $\frac{3.91 \times 10^{15}}{7.2 \times 10^{23}}$

Solution: This one's a little tricky. Remember how we multiply fractions: $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$. Reading this rule the other way, we can take something like $\frac{xw}{yz}$ and split it into $\frac{x}{y} \times \frac{w}{z}$. Now for the problem at hand:

$$\frac{3.91 \times 10^{15}}{7.2 \times 10^{23}} = \frac{3.91}{7.2} \times \frac{10^{15}}{10^{23}} = 0.543 \times 10^{-8} = \mathbf{5.43 \times 10^{-9}}$$

Homework

5. Calculate each product:

a. $(1.1 \times 10^6)(2.42 \times 10^4)$

b. $(2.0 \times 10^5)(3.7 \times 10^{-8})$

c. $(4.1 \times 10^6)(2.1 \times 10^{-4})$

d. $(1.9 \times 10^{-23})(2.3 \times 10^{-7})$

6. Calculate each quotient:

a. $\frac{4.42 \times 10^{15}}{1.7 \times 10^{13}}$

b. $\frac{8.0 \times 10^9}{2.0 \times 10^{20}}$

c. $\frac{9.2 \times 10^{-9}}{4.6 \times 10^4}$

d. $\frac{4.095 \times 10^{15}}{1.5 \times 10^{-8}}$

▣ *ADDING AND SUBTRACTING WITH SCIENTIFIC NOTATION*

▣ *APPLICATIONS*

EXAMPLE 6:

Light (and all electromagnetic radiation) travels in a vacuum at a speed of 3.0×10^8 m/s (meters /second). Find the distance (in meters) that light will travel in 7.32×10^7 seconds.



Solution: Since distance is the product of rate (speed) and time, we start with the formula $d = rt$, and then substitute the given numbers in the correct places:

$$d = rt$$

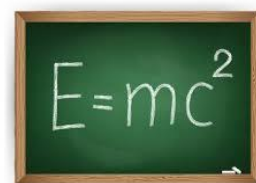
$$\Rightarrow d = (3.0 \times 10^8)(7.32 \times 10^7)$$

$$\Rightarrow d = (3.0 \times 7.32)(10^8 \times 10^7) \quad (\text{rearranging the factors})$$

$$\Rightarrow d = 12.96 \times 10^{15}$$

$$\Rightarrow \boxed{d = 1.296 \times 10^{16} \text{ m}}$$

EXAMPLE 7: Einstein's formula, $E = mc^2$, describes the equivalence of mass and energy. How much energy (in joules) is created when 1.45×10^7 kilograms of mass are converted to energy?



Solution: Start with the mass/energy formula, and plug in the numbers:

$$E = mc^2$$

$$\Rightarrow E = (1.45 \times 10^7)(3.0 \times 10^8)^2$$

$$\Rightarrow E = (1.45 \times 10^7)(9.0 \times 10^{16}) \quad [\text{from the rule: } (ab)^2 = a^2b^2]$$

$$\Rightarrow E = (1.45 \times 9.0)(10^7 \times 10^{16}) \quad (\text{rearranging the factors})$$

$$\Rightarrow \boxed{E = 1.305 \times 10^{23} \text{ J}}$$

EXAMPLE 8: Carbon 12 (C-12) is the isotope of carbon containing 6 neutrons in its nucleus. It is



known that 12 g of C-12 contains 6.02×10^{23} atoms. How many atoms are there in 24 g of C-12?

Solution: Since 12 grams contains 6.02×10^{23} atoms, and since 24 is twice 12, it follows that 24 grams must contain twice as many atoms:

$$\begin{aligned} \text{Number of atoms} &= 2(6.02 \times 10^{23}) \\ &= 12.04 \times 10^{23} \\ &= \boxed{1.204 \times 10^{24} \text{ atoms}} \end{aligned}$$

EXAMPLE 9: A spaceship travels at a rate of 23,500 mi/hr. How long (in hours) would it take to travel 150 million miles? Use *scientific notation* to perform all the calculations.



Solution: Since the problem refers to speed (rate) and distance, and asks for time, the formula $d = rt$ seems relevant. Solving this formula for t gives

$$t = \frac{d}{r}$$

Plugging in the given values gives

$$t = \frac{150 \text{ million}}{23,500}$$

$$= \frac{150,000,000}{23,500}$$

$$= \frac{1.5 \times 10^8}{2.35 \times 10^4}$$

$$= 0.638 \times 10^4$$

$$= 6.38 \times 10^3$$

It will take **6.38×10^3 hours** to travel 150 million miles.

Homework

7. How far (in meters, m) can light travel in the given amount of time?

a. 4.05×10^9 seconds

b. 7.3×10^{-5} seconds

c. 1.09×10^{11} seconds

d. 1 hour

8. For each given mass (in kilograms), use scientific notation to calculate the amount of energy created (in joules, J) when the mass is converted to energy:

a. 2.73×10^{14} kg

b. 5.09×10^{-12} kg

c. 500 kg

9. How many atoms of C-12 are there in the given mass?

a. 36 g of C-12

b. 120 g of C-12

c. 6 g of C-12

10. Find the travel time given the distance and the speed:

a. $d = 800$ million miles $r = 18,500$ mi/hr

b. $d = 1.7$ billion miles $r = 29,200$ mi/hr

c. $d = 8$ trillion miles $r = 95,000$ mi/hr

□ **LINKS**

Math is Fun

Solutions

1. a. Yes b. Yes c. No d. No e. No f. No
g. No h. No i. Yes j. Yes k. No l. No
2. a. 873 b. 209,000 c. 80 d. 23,600,000,000
e. 0.0599 f. 0.000601 g. 0.000001 h. 8.35
3. a. 9.2×10^6 b. 2.34×10^{10} c. 3.456×10^1
d. 1.23×10^{-5} e. 7.69967×10^4 f. 7.7×10^{-9}
4. a. 2.39×10^6 b. 1.2705×10^{16} c. 1.9×10^{-9}
d. 1.5×10^{-12} e. 8.3244×10^{-5} f. 4.0×10^{-8}
5. a. 2.662×10^{10} b. 7.4×10^{-3}
c. 8.61×10^2 d. 4.37×10^{-30}
6. a. 2.6×10^2 b. 4×10^{-11}
c. 2×10^{-13} d. 2.73×10^{23}
7. a. $1.215 \times 10^{18} \text{ m}$ b. $2.19 \times 10^4 \text{ m}$
c. $3.27 \times 10^{19} \text{ m}$ d. $1.08 \times 10^{12} \text{ m}$
8. a. $2.457 \times 10^{31} \text{ J}$ b. $4.581 \times 10^5 \text{ J}$ c. $4.5 \times 10^{19} \text{ J}$
9. a. $1.806 \times 10^{24} \text{ atoms}$ b. $6.02 \times 10^{24} \text{ atoms}$ c. $3.01 \times 10^{23} \text{ atoms}$
10. a. $4.32 \times 10^4 \text{ hrs}$ b. $5.82 \times 10^4 \text{ hrs}$ c. $8.42 \times 10^7 \text{ hrs}$

“What sculpture is
to a block of marble,
education is
to the human soul.”

Joseph Addison

